INFILTRATION INTO A SEASONALLY FROZEN SOIL AND MODELING OF SOIL FREEZING AND THAWING PHENOMENA

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Abstract

The process of soil freezing and thawing is desribed. Soil temperature, total soil moisture content and infiltration into a frozen sandy soil were measured at an experimental field station. A new method for solving the combined mass and heat balance equations was developed. The method is based on dividing the total energy into latent and sensible fractions so that the functional relationship between unfrozen water content and soil temperature is fulfilled. The model was tested against the results obtained at the experimental station.

1 Introduction

Frost has a prominent role in the hydrology of northern latitudes because of its effects on runoff phenomena. It also alters the infiltration characteristics of soils but it does not totally prevent the infiltration process during the snowmelt period.

As well as meteorological factors, snow depth and density, vegetation and soil characteristics also have an inluence on the formation and melting of soil frost.

In part I of this study infiltration into frozen soil was measured using a double ring infiltrometer, the neutron method and a lysimeter. In part II a mathematical model was developed

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to describe and simulate the process of soil freezing and thawing.

2 Factors affecting the infiltration capacity of frozen soil - a short literature review

According to several papers frozen soil is permeable. Infiltration capacity of frozen soil has been studied most in Canada and the Soviet Union (e.g. Kuznik and Bezmenov, 1963; Zavodchikov, 1962; Williams and Burt, 1974, ref. Nilsson, 1979.).In these studies the soil type being investigated was either clay or silt. The basic results obtained were:

- exact values of the infiltration capacity of different soils cannot be given, because infiltration depends on many factors
- soils frozen in a state of low water content are more permeable than soils which were wet before freezing
- the amount of fine material in the soil is inversely proportional to infiltration capacity
- in coarse textured soils the permeability decreases more rapidly at sub-zero temperature due to their lower content of unfrozen water
- if the frost depth is small the infiltrating water may melt the frost thus increasing the permeability rapidly

The infiltration capacity of frozen soil has been studied very little in Finnland. Some experiments were carried out in the 1930's by the Finnish Field Drainage Centre (Juusela, 1941). The soil was mainly composed of clay. As can be seen from Fig. 1, the outflow from subsurface drains followed very rapidly the changes in air temperature (and thus snowmelt) during spring 1935.

Seuna and Kauppi (1981) studied the influence of subdrainage on water quantity and quality in a cultivated area in southern Finland. The soil at the experimental station was composed mainly of heavy clay. During the snowmelt period 59% of the total runoff came from drains.

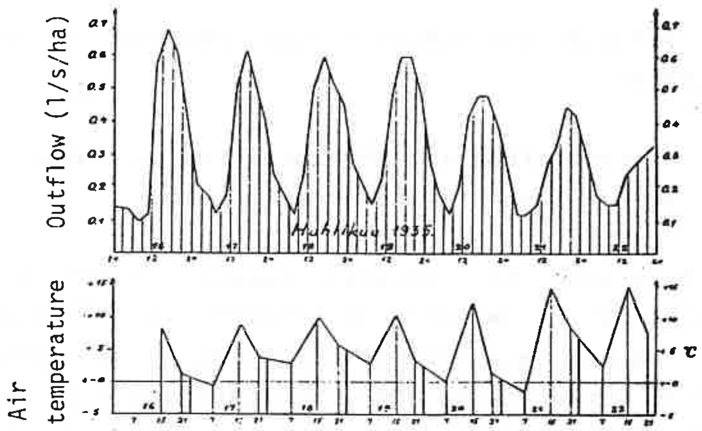

Fig. 1. Outflow from drains and air temperature in Jokioinen during 16 - 22.4. 1935 (Juusela, 1941).

During springs with deep soil frost the percentage of surface runoff was greater than average. In some years, when the upper layer of soil frost had already melted before the spring maximum runoff, the percentage of surface runoff was very small (Fig. 2.), although the total depth of the soil frost was still considerable.

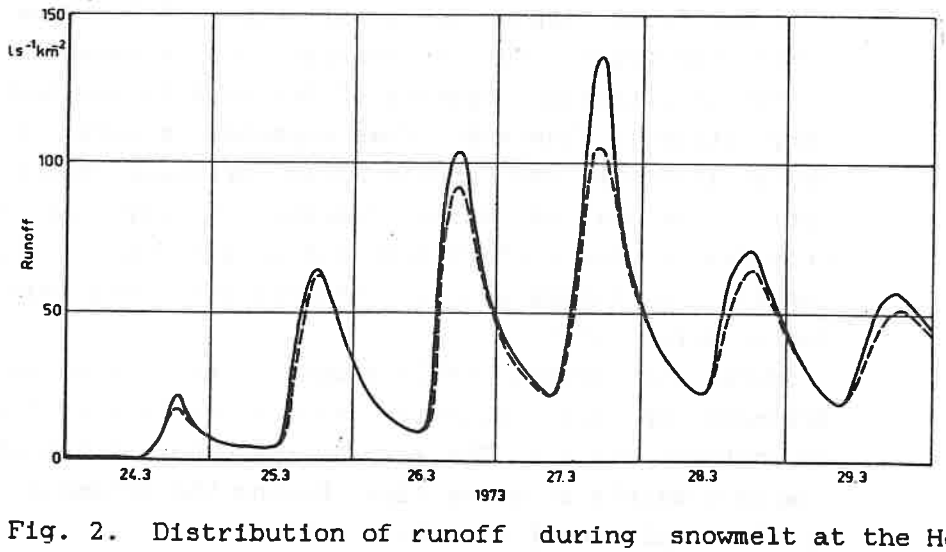

Fig. 2. Distribution of runoff during snowmelt at the Hovi basin in 1973. The solid line indicates total runoff and the dotted line outflow from subsurface drains (Seuna and Kauppi, 1981).

3 Experimental field station and measurements

The field station is situated in southern Finland (60°23' N and 25°02' E) on a glacifuvial delta formation. The dominant textural composition of the soils was sand and fine sand. The dominant vegetation at the station was lichen and heather.

The following measurements for the years 1969 - 1973 were used in this study: Air temperature, shortwave radiation, precipitation, snow thickness and density, snow and soil temperature, the depth of frost and changes in soil moisture at different depths. The grain size distribution, porosity, volume weight and pF-curves of the soil were also known. The mean depth to the groundwater level was generally over 6 m.

Snow temperatures at heights of 0, 2, 8, 12, 20, 30, 40, 50 and 60 cm and soil temperatures at depths of 1, 5, 20 40, 80, 150 and 250 cm were measured using copper-constantin thermocouples. The precision of the measurements was ±0.1°C. Methylene blue tubes were used to measure the depth of soil frost with the precision of 0.5 cm. The soil water content was measured along the vertical profile with a neutron probe at 10 cm intervals. This indirect way of determining soil moisture content was calibrated using gravimetric techniques (Lemmelä, 1970).

The infiltration measurements were made with a double ring infiltrometer (Lemmelä and Tattari, 1986). The percolation through a 1 m soil column was registered with a cubical 1 m^3 lysimeter. The soil moisture retention curves expressing the energy relations of soil water from pF 0 to pF 4.2 were measured according to the recommendations in "West European methods for soil structure determination" (1967), (Lemmelä, 1970.)

3.1 Infiltration into frozen soil

Examples of infiltration into a seasonally frozen ground as measured by double ring infiltrometers are shown in Fig. 3, as measured by soil moisture measurements in Fig. 4 and by the lysimeter in Fig. 5. The following conclusions can be drawn from the results (Lemmelä and Tattari, 1986). Frozen coarse-

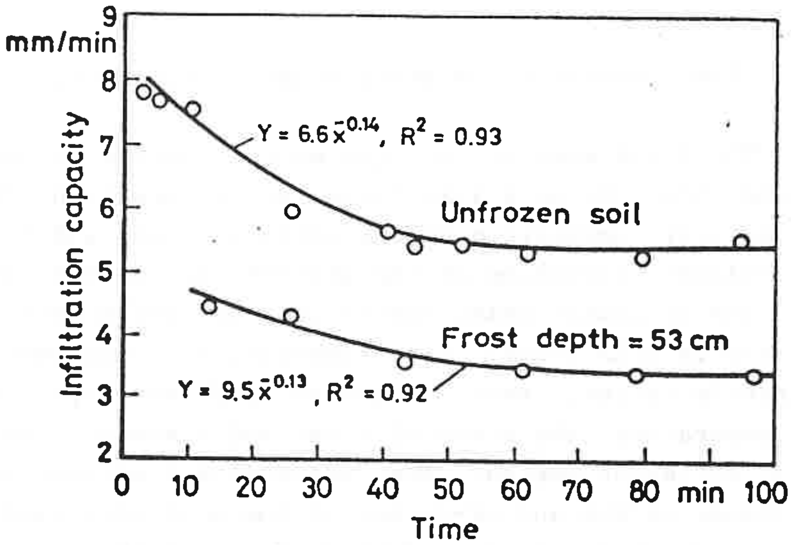

Fig. 3. The mean infiltration capacity of soil during the frozen and unfrozen periods (Lemmelä and Tattari, 1986).

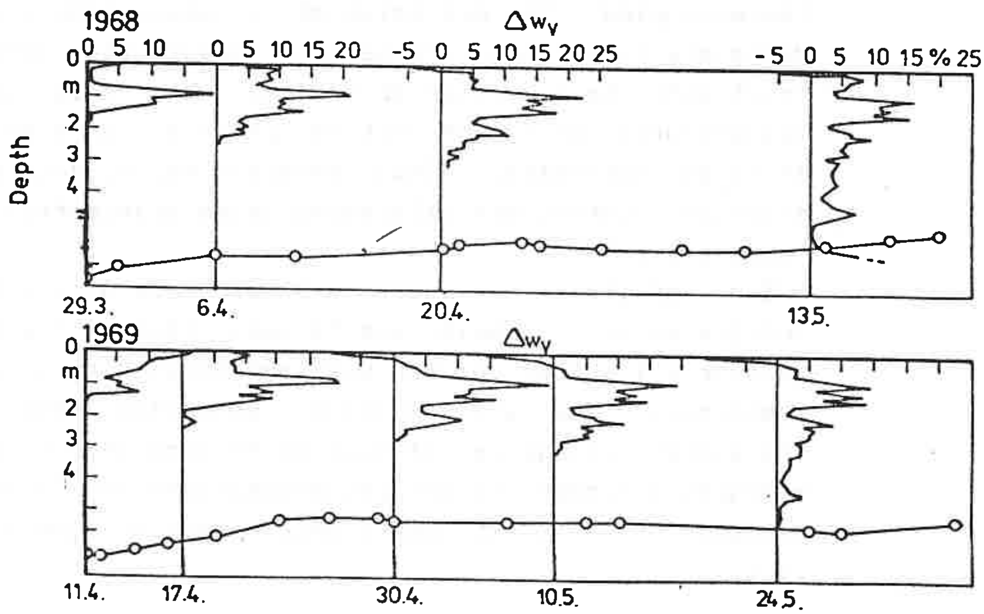

Fig. 4. Changes in soil moisture content[1] and groundwater level during the thawing periods in 1968 and 1969 (Lemmelä, 1970)

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[1] Comparison in soil moisture content is made with the soil moisture content just before the snow begins to melt.

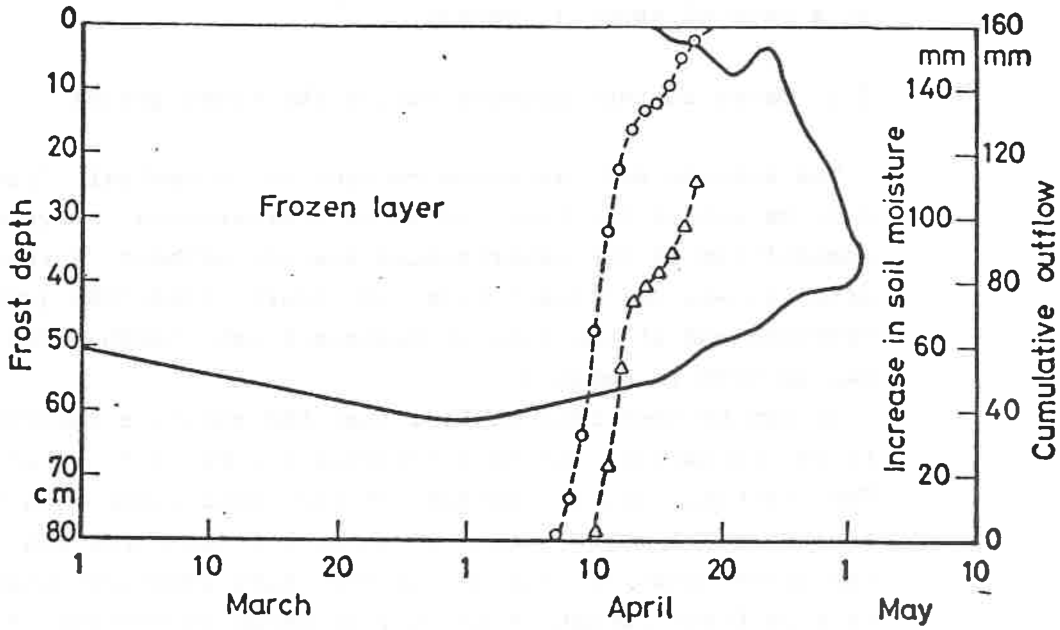


Fig. 5. Frost depth(▬▬), increase in soil moisture(o---o) and cumulative outflow (Δ---Δ) from the uppermost 1 m layer in spring 1969(Lemmelä and Tattari, 1986).

grained soils are relatively permeable and during the thaw period the final infiltration capacity greatly exceeds the maximum snowmelt intensity. According to ring infiltometer-measurements, the approximate mean value of the final infiltration capacity during the frost period was about 3 mm/min, the maximum daily snow melting varying from 8.4 to 29.3 mm. During the years from 1969 to 1973 the frost depth varied between 32 and 92 cm just after the snowmelt period. The increase in soil moisture content in the first 1 m layer varied between 70 and 135 mm and the cumulative outflow from the lysimeter varied between 31 and 74 mm before the frost layer started to melt from the surface.

According to the field measurements made in different spring seasons during the frost period, the depth of frost did not influence infiltration capacity as much as the initial water content of the frost layer.

The snowmelt progressed towards the groundwater surface at a rate of about 10 cm/day.

3.2 Water content changes during the frost period

The average soil moisture content in a typical frost layer 0-50 cm and in the layer 20-40 cm representing a typical soil composition at the experimental station without surface layer effects, was calculated from the years 1968-1984 just before freezing and at the date of maximum frost depth. The results can be seen in Table 1.

It can be seen from Table 1 that the moisture content during frost formation varied considerably , by 5.8 % at most. The initial water content of the soil layer when the soil freezes has a major influence on soil frost formation, because the latent heat of fusion is the most important heat budget term in frost formation as will be shown in Section 3.3. The moisture content did not change markedly during the frozen period. In the topmost 20 cm layer the measured moisture

Table 1. Average water content in two soil layers from 13 winters just before freezing and at the time of its maximum depth ($h_f = h_{f,max}$), their standard deviations and maximum and minimum values.

Layer cm	Water content w (vol. %)							
	Before freezing				$h_f=h_{f,max}$			
	w	st.dev	max	min	w	st.dev	max	min
0 -50	13.0	1.6	15.9	10.1	14.1	2.3	18.0	10.6
20-40	11.4	1.8	14.6	9.0	10.4	2.2	13.6	6.7

content increased but the reason for this was that no snow corrections were made to the water content measurements. The layer between 20 and 40 cm dries on average 1 % due to the movement of water downward under influence of gravitation.

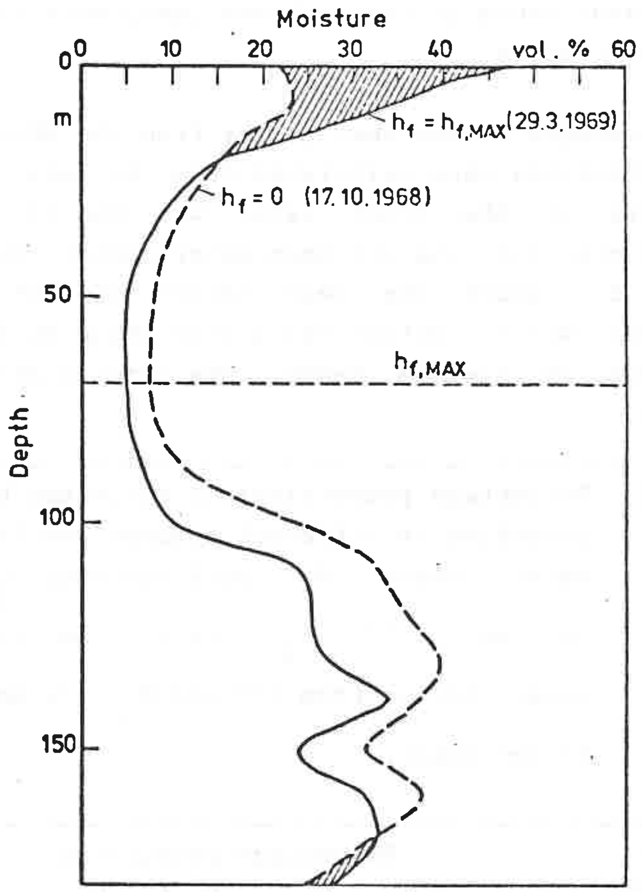


Fig. 6. Typical soil moisture curves before soil freezing and at the date of maximum frost depth

This phenomenon can also be seen in Fig. 6. The curves are from the winter of 1968-69, but they are typical for all the years.

Moisture changes were also calculated when groundfrost was at its maximum depth and when it had a value of maximum-10 cm in the layers $h_{f,max}$ -10 cm to $h_{f,max}$ and $h_{f,max}$ to $h_{f,max}$ +10 cm were also calculated. No moisture movement towards the freezing front was observed although theoretically a freezing-induced soil moisture redistribution should occur. The moisture content remained at a constant value in both layers.

3.3 Measurements of heat balance components during freezing and thawing

The averages of the heat losses from the whole frost layer during freezing were calculated from the data. The average thickness of the frost layer was 66 cm. Two different assumptions for the unfrozen water content were used (0.8 % and 1.5 % being the mean values in the calculations). The total water content had a mean value of 12.4 % when the frost had its maximum depth. The results are summarized in Table 2.

Table 2. Percentage proportions of different heat loss terms according to different assumptions for the unfrozen water content. H_1 = soil cooling, H_2 = water cooling to 0°C, H_3 = ice cooling from 0°C, H_4 = water cooling from 0°C and H_5 = latent heat of frozen water.

Unfozen	Percentage proportion					
water (%)	H_1	H_2	H_3	H_4	H_5	H
0.8	7.6	2.7	0.4	0	89.3	100
1.5	8.0	2.8	0.3	0.2	88.6	100

The latent heat had a dominant role,as can be seen from the table. It is also very important to know the amount of unfrozen water if the absolute values of changes in heat terms are to be calculated, because the difference in the amount of total energy losses reached 5.4 % when these two assumptions for unfrozen water content were compared.

The cold content of the frost layer when the frost was at its maximum depth was also calculated. This average cold content could freeze only about 1.8 mm of the meltwater as an average during these five years. The percentage proportions of this 1.8 mm were: layer 0-10 cmm 30 %, layer 0-50 cm 87 %

and layer 20-40 cm 30 % and 100 % for layer between 0 and 66 cm. The percentage of the soil itself in this cold content was about 80 %, and the percentages of ice and unfrozen water were 17 and 3 %, respectively.

4 Modeling of soil freezing and thawing phenomena

The possibility of numerically modeling the complex processes which occur in simultaneous heat- and soil-moisture transport in a freezing soil has received much attention during the past decade (e.g. Harlan, 1973; Guymon and Luthin, 1974; Taylor and Luthin, 1978; Guymon et al., 1980; Jansson and Halldin, 1980; Gilpin, 1980; Hromadka et al., 1981; O'Neill and Miller, 1982). Special attention has been devoted by Motovilov (1978, 1979) and Engelmark (1984) to the calculation of infiltration into a frozen soil.

4.1 Heat and water flow

The equations describing the combined heat and water flow are given by (1) and (2):

$$c_T \frac{\partial T}{\partial t} - \delta_i L \frac{\partial I}{\partial t} = \frac{\partial}{\partial z}\left(K_h \frac{\partial T}{\partial z}\right) - c_w q_w \frac{\partial T}{\partial z} \qquad (1)$$

$$C(\psi)\frac{\partial \psi}{\partial t} + \frac{\delta_i}{\delta_w} \cdot \frac{\partial I}{\partial t} = \frac{\partial}{\partial z}\left(K(\psi)\frac{\partial \psi}{\partial z} - K(\psi)\right) \qquad (2)$$

where z is a space coordinate, t is time, T is temperature, K_h is soil thermal conductivity, L is latent heat of fusion of water, c_T is volumetric specific heat of soil, the indices w and i refer to water and ice, respectively, δ is density, c is specific heat , q is flow of water, I is volumetric ice content, ψ is soil water potential, $C(\psi)$ is differential moisture capacity (a derivative of moisture content w with respect to potential ψ) and $K(\psi)$ is unsaturated hydraulic conductivity of the soil matrix.

A difficulty in the numerical solution of (1) and (2) is the inclusion of the ice term,since it generally dominates the solution as shown in Section 3.3. To avoid excessive numerical difficulties the assumption is often made that there exists a unique relationship between unfrozen water content w and soil temperature T in a frozen soil.

$$w = w(T) \quad ; T < 0°C \tag{3}$$

This means that all the water in soil will not freeze at zero temperatures. The lower the temperature the smaller will be the amount of unfrozen water. Since it is very difficult to measure the form of this function it is necessary to be able to predict it from other data that can be measured more easily. An assumption that is used in this study is that the functional form of (3) can be estimated from a known pF-curve. The validity of this estimate should be examined more carefully.

4.2 Freezing point depression in unsaturated soil

The formula expressing freezing point depression can be obtained thermodynamically from the generalized Clausius-Clapeyron equation (e.g. Miller, 1978):

$$u/\delta_w - u_i/\delta_i = (L/K)\Delta T \tag{4}$$

where u and u_i are pore water and pore ice pressure, respectively, K and ΔT are temperature in Kelvin and Celsius degrees. It can be assumed (e.g. Kinosita and Ishizaki, 1980) that ice pressure in unsaturated soil is zero. Then the freezing point depression is given by the following theoretical equation where u is expressed in centimetres. Thus u is equal to soil water potential.

$$\Delta T = \Psi/12200 \tag{5}$$

where T is in °C and potential in cm. If the pF-curve of the soil is known, a relationship between unfrozen water content and soil temperature (below °C) can be obtained from (5).

The freezing point depression curve calculated as a function of the soil water retention curve and based on equation (5) is shown in Fig. 7. For Bensby Silt the approximation is very good, but for Tomakomai Silt the estimated curve gives too low temperature values almost throughout the whole potential range. According to Kinosita and Ishizaki (1980) the reason for this is that some pressure acts on the soil-ice matrix owing to the adsorption of pore water to the surface of soil particles.

Unfortunately we did not find any other measured curves from the literature, so that the validity of equation (5) could not be tested thoroughly. Although it is quite obvious that equation (5) will not give fully accurate results in all cases it is used in the numerical calculations presented in Section 5. More research is necessary before freezing point depression can be estimated from generally measured quantities.

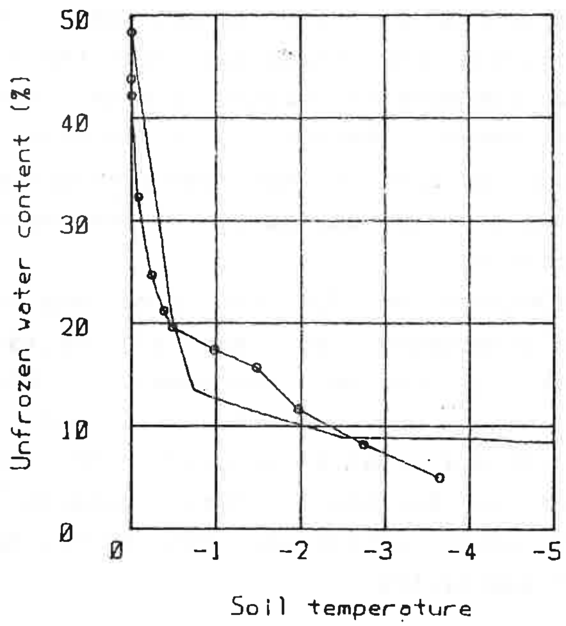
4.3 Energy concept for estimating the amount of unfrozen and frozen water content and soil temperature

Two basic alternatives exist for solving the combined heat- and mass transfer equations if a relationship between unfrozen water content and soil temperature is used. In the first version the derivative $\partial I/\partial t$ can be eliminated using the known $w = w(T)$ curve. This results in the so-called apparent heat capacity c_a, defined by

$$c_a = c_T + L \cdot \partial w/\partial T \qquad (6)$$

As was shown by Hromadka et al. (1981) the equations (1) and (2) can be combined into a single equation in a frozen zone. Engelmark (1984) has also used this concept. However, according to Guymon et al. (1980) numerical methods employing this approach require exceedingly small time steps, and small space discretization. Instability problems may result in lengthy simulations, involving time spans of a week or more.

On the basis of the foregoing experiments another method of solution was adopted. It was originally proposed by Karvonen (1986a). This method is based on the total energy



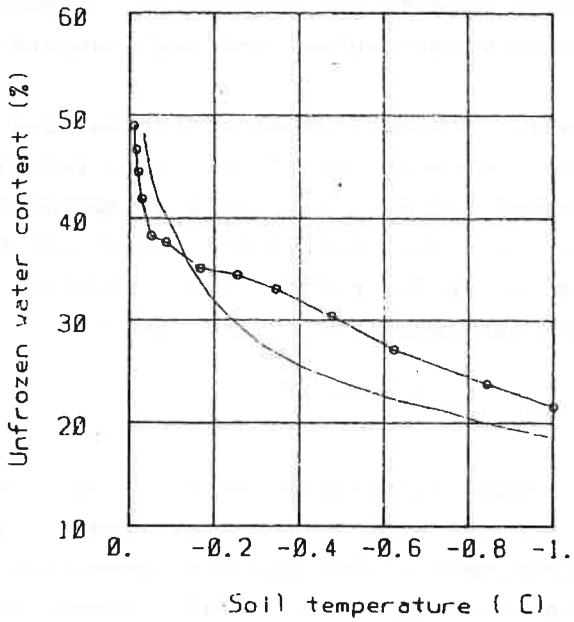


Fig. 7. Measured and estimated freezing point depression curves.(———) Measured. (o---o) Calculated based on equation (5) and the measured pF-curve.
a) Bensby silt (Engelmark, 1984.)
b) Tomakomai silt (Kinosita and Ishizaki, 1980).

concept, in which the total energy is divided into latent and sensible fractions using the known relationship between unfrozen water content and soil temperature. Consider the total energy needed to raise the temperature of a soil sample to 0°C from a negative temperature. Moreover, it is assumed that the total water content is known,whereas both the amount of frozen and frozen water and the soil temperature are not known. Then the energy E needed can be expressed as

$$E = - f_s \cdot c_s \cdot T - w \cdot c_w \cdot T - I \cdot c_i \cdot T + L \cdot I \qquad (7)$$

$$= - f_s \cdot c_s \cdot T - w \cdot c_w \cdot T - (W-w) \cdot c_i \cdot T + L \cdot (W-w)$$

where the subindices s, w and i refer to soil, water and ice, respectively, f is the volumetric fraction of soil, w is the unfrozen water content, I is the ice content, W is total water content (water + ice) and c the specific heat of different constituents. In equation (7) the amount of ice is expressed as the amount of frozen water (i.e. the volumetric amount of ice is about 9 % greater).

Equation (7) can be used in the model when the unknown unfrozen water content and soil temperature are calculated using known values of total water content W and energy state E. Equation (7) contains only two unknowns (w and T) and if the relationship w=w(T) is used, there exist two equations for solving the unknowns. The iterative solution is obtained by the Newton-Raphson method. The methods of obtaining the total water content W and the energy state E are described in Section 4.4.

4.4 Numerical solution method

The proposed model solves the mass and energy balances using the finite element method. This method was selected because different types of boundary conditions can easily be adopted into this technique.

The actual solution proceeds through a three-stage process.

1) Solve the mass balance equation assuming no change in the ice content (the total amount of water W is thus obtained).

2) Solve the heat balance equation neglecting the latent part. By solving the new temperature profile the amount of energy lost from the system can be calculated (i.e. the energy state E can be calculated).
3) Solve the actual unfrozen water content w and ice content I as described in Section 4.3.

Two possibilities exist when iterating during one time step. In the first version stages 1) and 2) are solved and after that w and I are solved without further iterations. In the second version stages 1)-3) are repeated iteratively until convergence is attained. In calculating the energy state the amount of latent heat generated during the time step is taken into account. The first version can be considered as explicit with respect to the calculation of the energy state of the system. It is preferable when the simulation of long time intervals is needed.

4.5 Initial and boundary conditions

The intial state of the system must be known at time t= 0. This implies that either the soil moisture content or total hydraulic head (e.g. steady-state situation) must be given for each nodal point for the solution of the mass balance equation. Moreover, it is necessary to define the initial temperature profile.

Different types of boundary conditions can be used,depending on the available data. At the upper boundary, the maximum possible infiltration capacity (rainfall + snowmelt) is prescribed. In the numerical solution the true infiltration capacity is maximized by a method proposed by Neuman et al. (1974). The upper boundary condition for the heat equation is the measured soil surface temperature, or if this is not measured then the air temperature can be used.

At the lower boundary, three types of boundary conditions can be used: prescribed groundwater level (outflow from the system is calculated), no-flow boundary (groundwater level calculated) or outflow to parallel drains calculated by the Ernst (or Hooghoudt) formula (depth of groundwater level is calculated). The latter alternative can be considered to be a quasi-two-dimensional solution in the case of a system where drain spacing and depth are given and groundwater level is the variable to be calculated. In the heat flow equation temperature or zero heat-flow can be used as the lower boundary condition.

4.6 Input and output data

Again, different types of input data can be used, if available. The minimum meteorological input data needed are precipitation and air temperature. Moreover, it is necessary to have the pF-curves of the soil and the saturated hydraulic conductivities, thermal conductivities and specific heat values for soil, water and ice. The unsaturated hydraulic conductivity is calculated by a method originally proposed by Sigvard Andersson (1969) and slightly modified by Karvonen (1986b). According to a comparison made with 30 measured curves the Andersson method gave at least as good results as the well-known Mualem method (1976).

The primary aim of the model is to simulate the effects of soil heat extraction on an annual scale in a case where the depth and spacing of subsurface drains are given. The most important output data needed are soil moisture, groundwater level and soil temperature. With the model it is possible to compare e.g. the effect of different drain spacing (and/or depth) on soil moisture and soil temperature. This model can be used together with a potential crop production model to calculate the actual yield when drain spacing and depth (and the necessary soil data) are known (Karvonen, 1986c).

The additional output data from the model include e.g. the outflow to the drains and soil trafficability criteria.

5. Numerical experiments

5.1 Experiment 1

The model was tested against the experiment carried out by Kalyuzhnyi et al. (1984). The column was 50 cm long. Measured temperature were given both as an upper and a lower boundary condition in the heat balance equation. No-flow boundary was used an an upper boundary condition in the mass flow equation. At the lower boundary constant hydraulic head was given. As a result of flow of water to the freezing front sharp increase in the total water content was observed (e.g. from 4.5 % to 14.1 % at the depth of 16 cm, see also Fig. 9).

Changes in temperature and water content were measured during soil freezing. The soil moisture retention curve was estimated from measured steady-state initial profile. The unsaturated hydraulic conductivity was calculated using the method proposed originally by Andersson (Karvonen, 1986b). The thermal conductivity of frozen soil was estimated using formulas given by Kersten (1949) (ref. Jansson and Halldin, 1980). The objective of this simulation example is two-fold. First, the applicability of the model to simulate freezing induced soil moisture redistribution was tested. Second, the sensitivity of the solution to small variations in the unsaturated hydraulic conductivity values was evaluated.

The results of the soil temperature simulations are shown in Fig. 8, and measured and computed total water content in Fig. 9 and 10. However, since the soil temperature was defined both at the upper and lower boundary condition the deviation between measured and calculated values should be very small. In this respect the simulation results can be considered to be only satisfactory.

The simulation of total water content is uncertain as can be seen from the results. The system is very sensitive to small variations in the unsaturated hydraulic conductivity. The original curve (B in Fig. 11) gave too small increase in the total water content at the freezing front (see Fig. 10). When slightly higher values (curve A in Fig. 11) were used the simulated soil moisture redistribution is very near the measured one (Fig. 9 and 10). It is obvious that the most important parameter in the combined heat and mass transfer is the unsaturated hydraulic conductivity. Unfortunately the shape of this function is very difficult to measure and it is usually estimated indirectly from measured pF-curve.

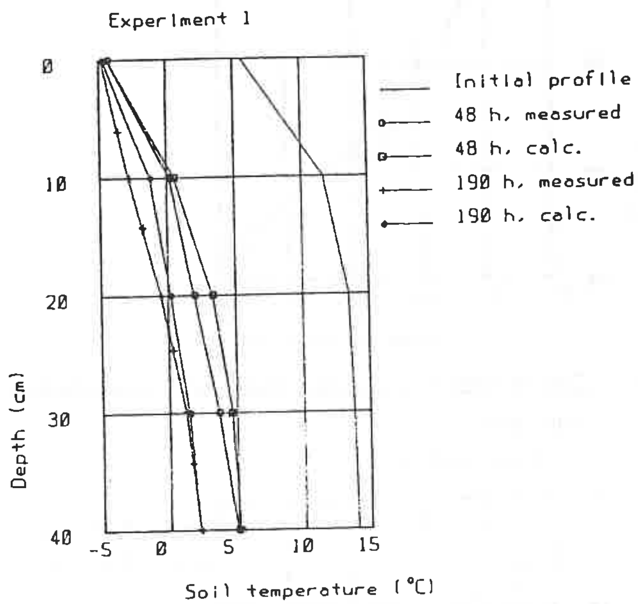


Fig. 8. Experiment 1, measured and calculated soil temperature.

5.2 Experiment 2

The second simulation example is from the experimental station referred already in Section 3. The calculated and measured cumulative outflow from the uppermost 1 m layer in spring 1969 are shown in Fig. 12. In the simulations the lower boundary condition was defined as a flux-boundary. The outflow from the system was calculated assuming a unit gradient (i.e. outflow equals the hydraulic conductivity of the lowest node). The simulations are in quite good agreement with the measured values. The depth of soil frost during the simulation interval is shown in Fig. 5.

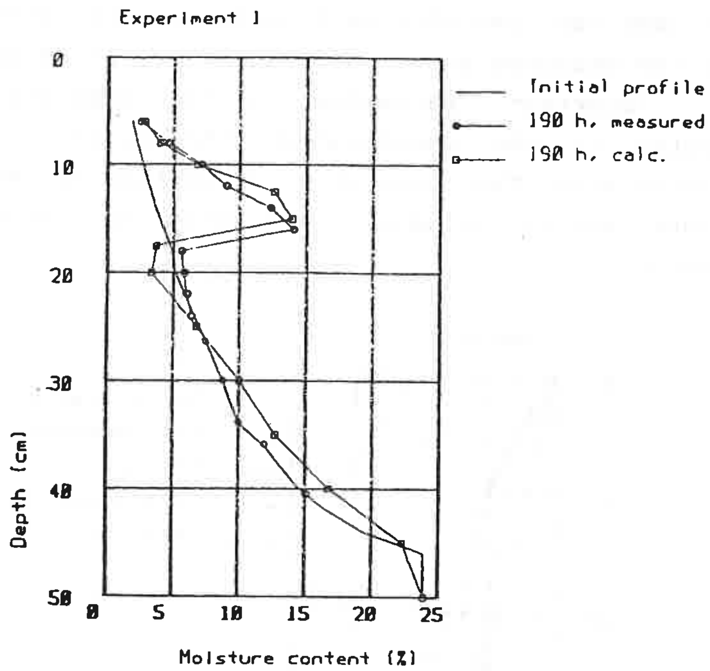


Fig. 9. Experiment 1, measured and simulated total water content.

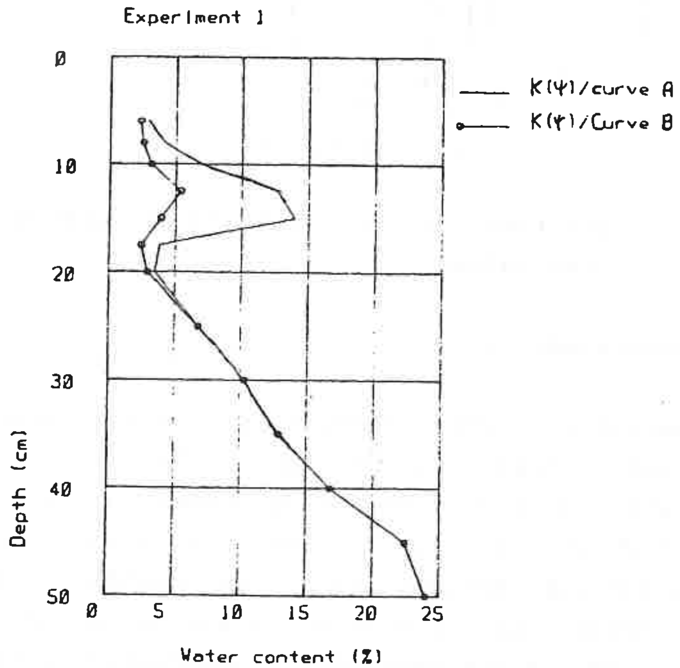


Fig. 10. Experiment 1, simulated water content using two different curves for unsaturated hydraulic conductivity.

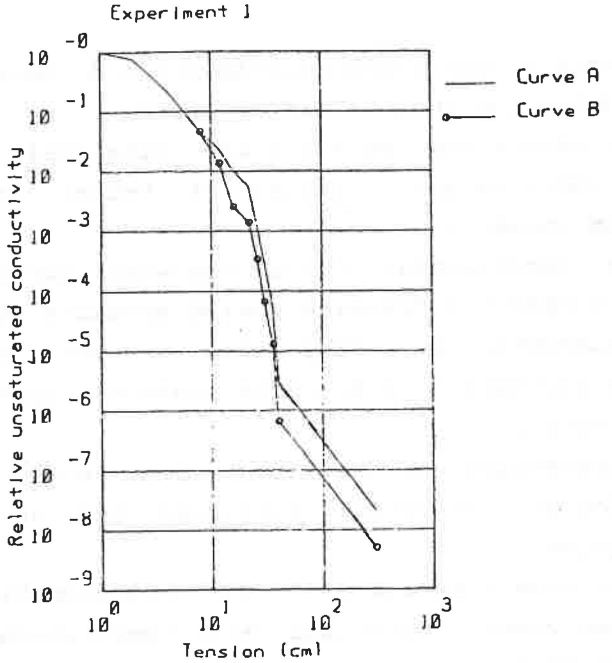


Fig. 11. Experiment 1, two different unsaturated hydraulic conductivity functions.

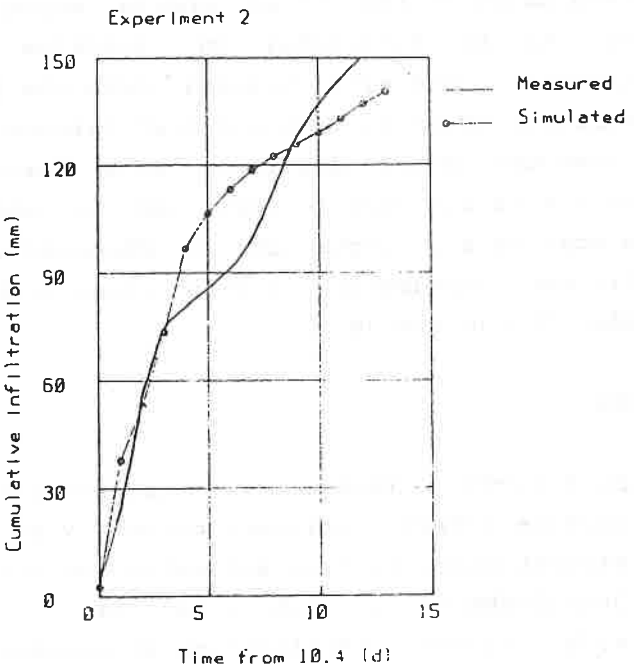


Fig. 12. Calculated and measured cumulative outflow from the uppermost 1 m layer in spring 1969.

6. Conclusions

On the basis of the field experiments and computer simulations, the following conclusions can be drawn:

1° Frozen coarse-grained soils are relatively permeable and the infiltration capacity of frozen soil exceeds the maximum snowmelt.

2° At the experimental field the water content changes were small during the freezing period probably due to the depth of groundwater level (the original soil water potential was so low that soil freezing could not draw water to the frost zone).

3° The percentage of the latent heat (phase change of water into ice or vice versa) dominates the energy balance of the system.

4° An approximate estimate of the relationship between the unfrozen water content and soil temperature below zero can be obtained on the basis of the measured soil water retention curve and equation (5). More research is needed to verify this assumption.

5° A method based on the total energy state of the system proved to be successful in avoiding the numerical difficulties generally arising from the inclusion of ice accumulation terms in mass and heat balance equations.

6° The computer model developed is a reasonable basis for larger simulation model that can be used in optimizing drain spacing and depth and in choosing optimum drainage coefficient (naturally, a crop production model must be included in the system).

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