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THE USE OF RAINFALL-RUNOFF MODELS IN REAL TIME FORECASTING AND CONTROL

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INTRODUCTION

Studies on rainfall-runoff modelling have been carried out in the Laboratory of Hydrology and Water Resources Engineering, Helsinki University of Technology since 1978. Vakkilainen and Karvonen (1980) and Karvonen (1980) have studied the use of the SSARR model in a research programme which also included the application of linear rainfall-runoff models (Järvinen, 1982) and snowmelt-runoff models (Hiitiö, 1982). Nyrhinen (1982) and Rajantie (1986) have studied the applicability of the Swedish HBV-model. Vakkilainen and Karvonen (1982) have developed the SATT model. In this model the soil moisture can be treated in a physically based way or using a conceptual approach. The flood routing, as well, can be calculated either by using the Saint Venant equations or the Muskingum method. The model includes a simple adaptive system which improves the accuracy of the forecasts when new measured data is available. Karvonen (1985) formulated both the HBV model and the SATT model within the framework of state-space analysis and used the extended Kalman filter for improving the forecasts.

For the optimal control of a watercourse an optimization method is needed, too. Both linear and dynamic programming models have been developed for operating a single lake or reservoir (Vakkilainen, 1978; Heikkinen, 1982).

The rainfall-runoff models mentioned previously have proved to be either too complicated to use together with control methods or the correction capability of these models is insufficient. Hence, the applicability of transfer function models has been evaluated (Malve, 1986). It is also difficult to use linear and dynamic programming techniques for controlling a multi-reservoir systems. Therefore, a method based on the Pontryagin maximum principle has been recently developed (Karvonen, 1986). In this paper these two studies have been shortly reviewed.

REAL TIME FORECASTING

The purpose of the work of Malve (1986) was to combine and test conceptual rainfall-runoff model and time series models (AR-models and transfer function models). Model combinations were simple in order to obtain suitable models for real time forecasting. The main emphasis was to compare the predictive ability of three different models:

- 1) HBV-model
- 2) HBV-model + AR-model
- 3) Transfer function models

The AR-model used together with the HBV-model was aimed at modeling the residual between the measured and forecasted discharge, i.e. the residuals of one or two previous days were used to correct the forecasted values. In model version 3) the rainfall losses were estimated using the soil moisture storage of the HBV-model.

The model combinations were tested using observations from four catchments in western and northern Finland: Tujuoja ($A=21 \text{ km}^2$), Yläneenjoki (195 km^2), Loimijoki (1980 km^2) and Ounasjoki (12300 km^2). Transfer function models proved to be the best ones (Malve 1986). The AR-model improved clearly the short-term forecasting capability of the HBV-model. A brief review of the results is shown in Table 1.

Examples of the final equations of model version 3) for the Loimijoki catchment are:

$$Q(k+1)=0.6477*Q(k)+0.0935*P(k)+0.0923*P(k-1)+1.3677*Y(k)$$

$$Q(k+1)=0.6374*Q(k)+3.1691*Y(k)$$

where $Q(k+1)$ and $Q(k)$ are discharges at time $k+1$ and k , respectively. $P(k)$ and $P(k-1)$ are the effective rainfall rates at two previous days and $Y(k)$ is the discharge of a nearby smaller reference catchment.

In Fig. 1 a comparison of the calculated and measured discharges using model versions 1) and 3) in the Ounasjoki watershed is presented.

Table 1. The comparison of the results of different models

Watershed	Model 1)	Model 2)	Model 3)
Tujuoja	0.85	0.93	0.93
Yläneenjoki	0.79	0.87	0.91
Loimijoki	0.84	0.92	0.93
Ounasjoki	0.81	0.98	0.99

The goodness of fit is calculated using the equations:

$$RR = (F_0 - F)/F$$

$$F_0 = (Q(i) - Q_M)^2$$

$$F = (Q(i) - Q_C(i))^2$$

where $Q_C(i)$ and $Q(i)$ are calculated and measured discharge, respectively, and Q_M is the mean of measured discharges.

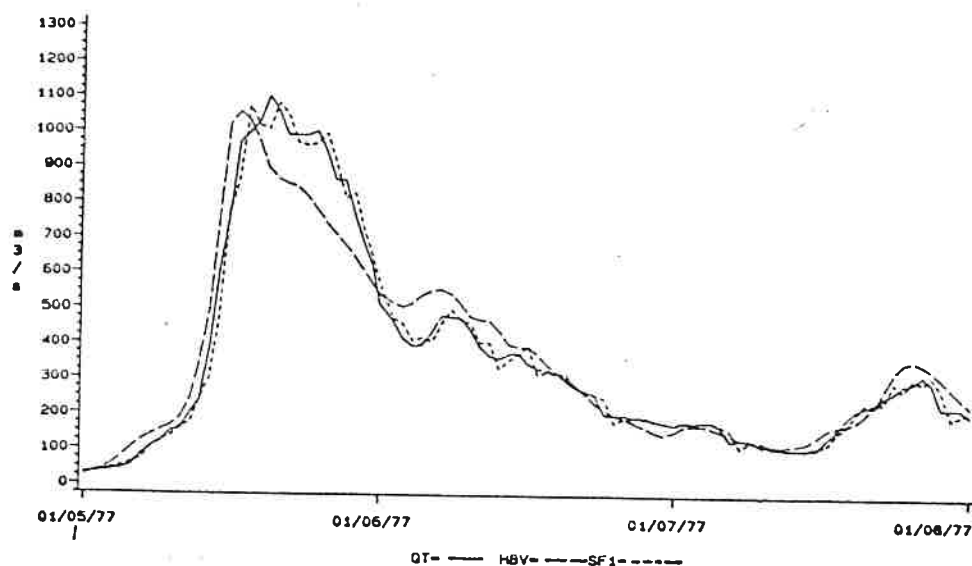


Fig. 1. Measured and calculated discharge in the Ounasjoki catchment area. QT is measured discharge, HBV and SF1 are calculated discharges using the HBV-model and transfer function model, respectively.

REAL TIME CONTROL OF MULTI-RESERVOIR SYSTEM

The optimal control system is based on the Pontryagin maximum principle (e. g. Kalaba, 1982). The general idea of the maximum principle is to maximize (or minimize) the value of the selected cost function without violating the constraints (maximum and minimum discharge and water level). The cost function can be e.g. to maximize the energy product of the reservoir system or to minimize the outflow of the reservoir system in order to prevent damages caused by floods.

The Kalajoki reservoir system (see Fig. 2 and 3) has been used as a testing example. The total area of the watershed is 4 200 km². In the area there are five regulated reservoirs which can be used for real time control of river flow. Inflows into each reservoir are predicted using transfer function models. Two types of forecasts are used. First, short-term forecast for 5 to 10 days are calculated using predicted air temperature and precipitation forecasts. Second, the total volume of the spring flood is estimated using meteorological data of previous years.

A model of the reservoir system is needed in the optimization. The mathematical model of the reservoir system is composed of waterbalance equations and transfer function models. The last mentioned are used to describe the flow in rivers between the reservoirs. In the optimization the goal is to minimize outflows from the reservoirs.

The general idea in real time operation of the reservoirs can be summarized as follows:

- 1) Short-term and long-term inflows are forecasted. The long-term forecasts (total inflow volume) are needed to prevent the filling of the reservoirs too early.
- 2) The Pontryagin maximum principle is used to obtain optimal outflow trajectories for each reservoir for the whole operating period (e.g. 2 weeks - 2 months).
- 3) The operation policy is fulfilled until new measurements are available.
- 4) These new measurements are used to update the inflow forecasting

models. The procedure is repeated from the step 1).

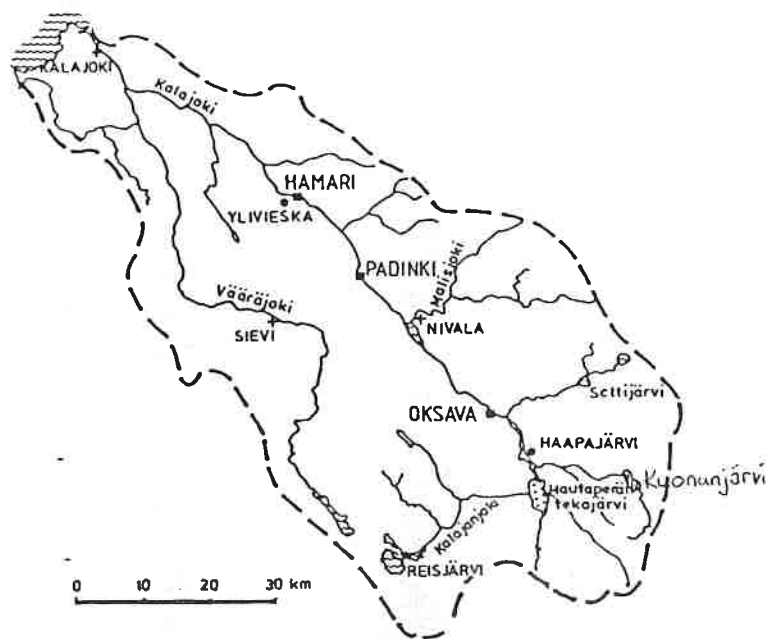
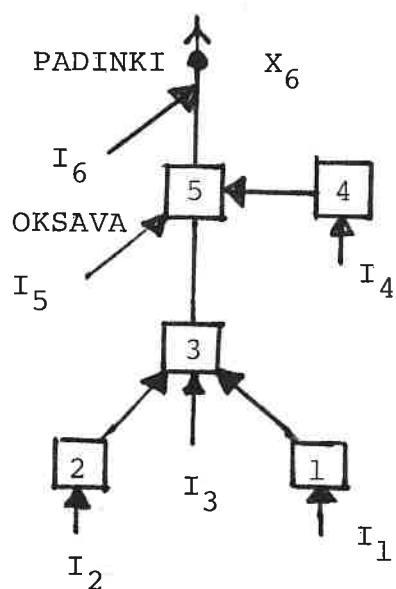


Fig. 2. The Kalajoki watershed area.



Nro	Reservoir
1	Kuonanjärvi
2	Reis-Vuhtojärvi
3	Hautaperä
4	Settijärvi
5	Haapajärvi

Fig.3. The reservoirs of the Kalajoki watershed area.

An example of the optimized outflow trajectories are presented in Fig. 4. In this figure the inflows into each reservoir are shown, too.

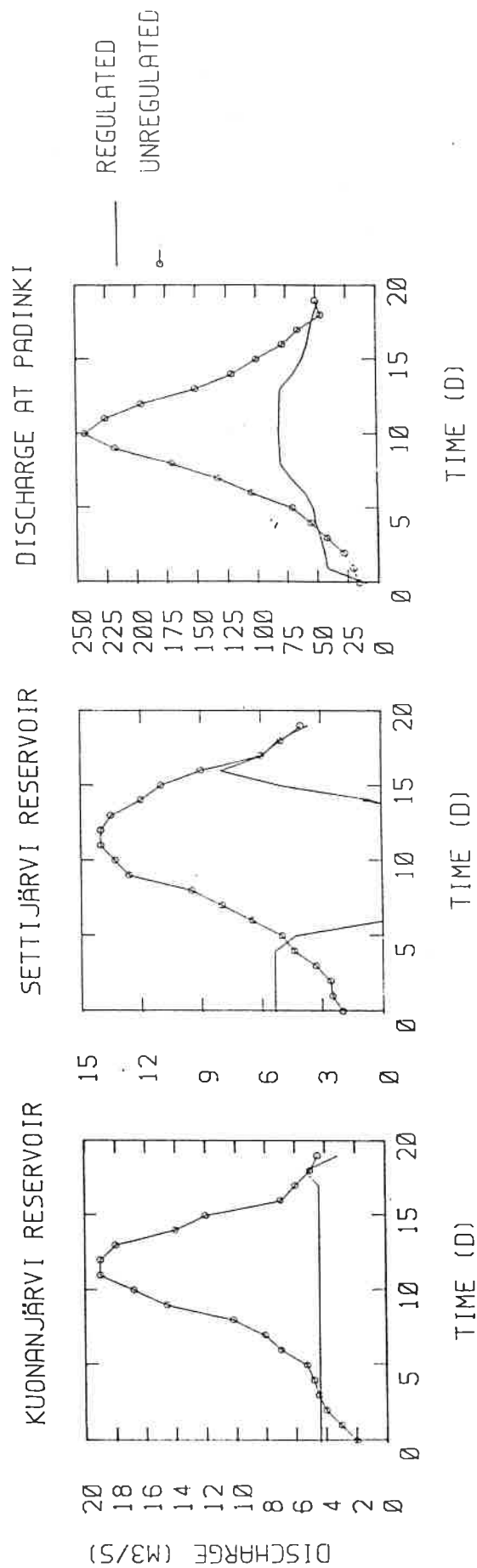
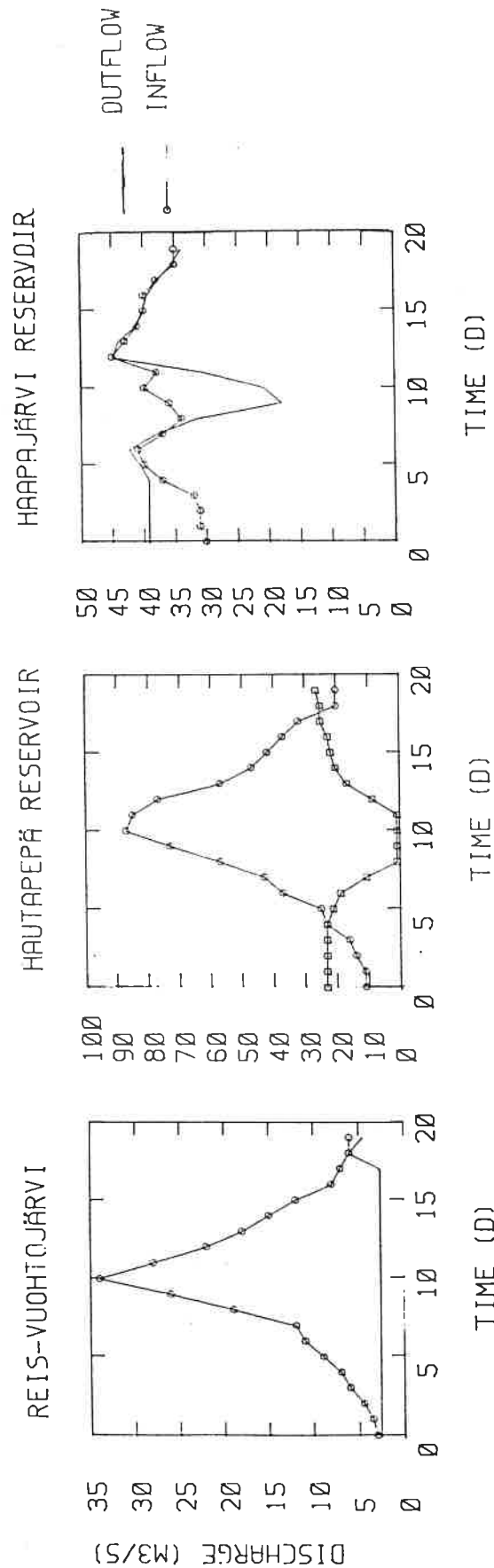


Fig. 4. Optimal outflow trajectories for each reservoir and the regulated discharge at Padinki.

CONCLUSIONS

Transfer function models have proved to be a powerful tool in real time flood forecasting. In the context of these type of models it is easy to accomplish the updating of the flow forecasts using the latest hydrological data. Moreover, the information of the experimental and representative basins can be included in the forecasting model.

The Pontryagin maximum principle seems to be applicable to optimal control of regulated reservoirs. The formulation of the objective function is perhaps the most difficult part of the optimization procedure.

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