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A model for simulating freezing and thawing of unsaturated soils

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Abstract

A model for simulating ground freezing and thawing has been developed. The model is based on the numerical solution of combined mass and heat transfer equations. The partial differential equations are solved using finite element method. Comparison of calculated values with field measurement showed that the accurary of the model results is sufficient.

1 INTRODUCTION

The possibility of numerical modelling of the complex processes which occur in simultaneous heat- and soil-moisture transport in a freezing soil has received much attention during the past decade (e.q. Harlan, 1973; Guymon and Luthin, 1974; Taylor and Luthin, 1978; Guymon et al., 1980; Jansson and Halldin, 1980; Hromadka et al., 1981; O'Neill and Miller,1982).Special attention has been devoted by Motovilov (1977, 1978, 1979) and Engelmark (1984, 1986) to the calculation of infiltration into a frozen soil.

During the years 1985-1988 a research program regarding ground freezing and thawing was carried out in the Laboratory of Hydrology and Water Resources Management at Helsinki University of Technology. The primary goal was to develop a model for computing the whole freezing and thawing cycle in field conditions. The model is intented to be part of a larger rainfall-runoff model and therefore it is necessary to have a solution method where time steps of few hours can be used. Because of the use of the model is in the field of hydrology, frost heave is not included. In this article only the main features of the model are described. The comprehensive description has been given by Karvonen (1988).

2 NUMERICAL SOLUTION OF COMBINED MASS AND HEAT FLOW IN THE SEASONALLY FROZEN SOIL

2.1 Combined heat and mass transfer

The equations describing the combined heat and water flow are given by (1) and (2):

$$C_s \frac{\partial T}{\partial t} - d_i L \frac{\partial I}{\partial t} = \frac{\partial}{\partial z} [K_T(w) \frac{\partial T}{\partial z}] - c_w q_w \frac{\partial T}{\partial z} \quad (1)$$

$$C(h) \frac{\partial h}{\partial t} + \frac{d_i}{d_w} \frac{\partial I}{\partial t} = \frac{\partial}{\partial z} [K(h) \frac{\partial h}{\partial z} - K(h)] - S(h) \quad (2)$$

where z is a space coordinate (m), t is time (d), T is temperature (°C), $K_T(w)$ is the soil thermal conductivity (W m^{-1} $°C^{-1}$), L is latent heat of melting of water at T=0 °C (J kg^{-1}), C_s volumetric specific heat of soil (J m_{-1} $°C^{-1}$), d_i and d_w are density of ice and water (kgm^{-1}), respectively, c_w is specific heat of water (J kg^{-1} $°C^{-1}$), q(w) is flow of water (m d^{-1}), I is volumetric ice content, h is soil water potential, C(h) is differential moisture capacity (m^{-1}) and K(h) is unsaturated hydraulic conductivity of the soil matrix (m d^{-1}) and S(h) the sink term representing the volume of water taken up by the roots.

A difficulty in the numerical solution of (1) and (2) is the inclusion of the ice term, since it generally dominates the solution (e.q. Guymon et al. 1980). To avoid excessive numerical difficulties the assumption is often made that there exists a unique relationship between unfrozen water content w and soil temperature T in a frozen soil.

$$w = w(T); \quad T < 0\,^{\circ}C \qquad (3)$$

This implies that all the water in soil will not freeze at zero temperatures.The lower the temperature the smaller will be the amount of unfrozen water. Since it is very difficult to measure the form of this function it is necessary to be able to predict it from other data that can be measured more easily.

2.2 Freezing point depression in unsaturated soil

The formula expressing freezing point depression ΔT can be obtained (e.q. Kinosita and Ishizaki, 1980):

$$\Delta T = 0.08\, \Delta pw \qquad (4)$$

where the unit of pw is kg m^{-2} (pw has a negative value).In this paper it has been assumed that pw corresponds to the soil water potential h expressed in units m. Hence, equation (4) can be given as follows:

$$T = h / 122 \approx 0.008h \qquad (5)$$

where T is in $^{\circ}C$ and potential h in m.If the pF-curve of the soil is known, a relationship between unfrozen water content and soil temperature (below 0 $^{\circ}C$) can be obtained from (5). According to Miller and Dirksen (1966), T was 0.004h for the soil in which particles contact directly with each other and 0.009h for the soil in which particles contact which each other through a water film absorbed on the surface of soil particles. Therefore, in the numerical simulations one has to choose the value for temperature depression between 0.004h - 0.008h (0.009h).

2.3 Estimation of the amount of unfrozen and frozen water content and soil temperature

Two basic alternatives exist for solving the combined heat- and mass transfer equations if a relationship between unfrozen water content and soil temperature is used. In the first version the derivative $\partial I/\partial t$ can be eliminated using the known $w = w(T)$ curve. This results in the so-called apparent heat capacity C_a, defined by

$$C_a = C_s + L\frac{\partial w}{\partial T} \qquad (6)$$

As was shown by Hromadka et al.(1981), equations (1) and (2) can be combined into a single equation in a frozen zone. Engelmark (1984, 1986) has also used this concept. However, according to Guymon et al.(1980) numerical methods employing this approach require exceedingly small time steps, and small space discretization. Instability problems may result in lengthy simulations, involving time spans of a week or more. According to Engelmark (1986) the maximum time step with a space discretization of 2.5 cm was 8 seconds. The total simulation time for a complete freezing-thawing cycle using this approach requires much computer time.

A method based on the total energy concept is used in the simulation program (Karvonen 1988). In this approach the total energy is divided into latent and sensible fractions using the known relationship between unfrozen water content and soil temperature.

Consider the total energy needed to raise the temperature of a soil sample to 0 °C from a negative temperature if the total water content is known, whereas both the amount of unfrozen and frozen water content and the soil temperature are not known. Then the energy E needed can be expressed as

$$E = -f_s \cdot c_s \cdot T - w \cdot c_w \cdot T - I \cdot c_i \cdot T + L \cdot I$$
$$= -f_s \cdot c_s \cdot T - w \cdot c_w \cdot T - (W - w) \cdot c_i \cdot T + L \cdot (W\text{-}w) \tag{7}$$

where c_s, c_w and c_i are the specific heat of soil, water and ice, respectively, f_s is the volumetric fraction of soil, w is the unfrozen water content, I is the ice content, W is total water content (water + ice).In equation (7) the amount of ice is expressed as the amount of frozen water (i.e. the volumetric amount of ice is about 9 % greater).

Equation (7) can be used in the model for calculating the unknown unfrozen and frozen water content and soil temperature using known values of total water content W and energy state E. Equation (7) contains only two unknowns (w and T) and if the relationship w=w(T) is used, there exist two equations for solving the unknowns. The iterative solution is obtained by the Newton-Raphson method.

2.4 Numerical solution method

The proposed model solves the mass and energy balances using the finite element method. The actual solution proceeds through a three-stage process:

1) Solve the mass balance equation assuming no change in the ice content (the total amount of water W is thus obtained).
2) Solve the heat balance equation neglecting the latent part.By solving the new temperature profile the amount of energy lost from the system can be calculated (i.e. the energy state E can be calculated).
3) Solve the actual unfrozen water content w and ice content I as described in Section 2.3.

2.5 Initial and boundary conditions

The initial state of the system must be known at time t = 0. This implies that either the soil moisture content or total hydraulic head (e.q. steady-state situation) must be given for each nodal point for the solution of the mass balance equation. Moreover, it is necessary to define the initial temperature profile.Different types of boundary conditions can be used, depending on the available data. The upper boundary conditions of the mass balance equation is usually governed by meteorological conditions (snowmelt can be included). At lower boundary either a prescribed groundwater level or a zero-flux or computed flow to sub-surface drains can be used.

The upper boundary conditions for the heat equation is the measured soil surface temperature,or if this is not measured then the air temperature measured at the level of e.q. 2 m can be used.In the heat flow equation prescribed temperature or zero heat-flow can be used as the lower boundary condition.

When the depth of snow cover has been calculated or given as input data, the estimation of the soil surface temperature below the snow-cover can be carried out by equating two energy fluxes (e.q. Jansson and Halldin, 1980):

1) Steady state flux between the soil surface and air, i.e.
   through the snowpack
2) Steady state flux between the first nodal point below the soil surface and the soil surface

$$K_{snow} \frac{T_{air} - T_{surf}}{z_{snow}} = K_{soil} \frac{T_{surf} - T_{old}}{L_1} \qquad (8)$$

where K_{snow} is the thermal conductivity of snow (W m^{-1}

°C^{-1}), K_{soil} is the thermal conductivity of the soil, T_{surf} is the unknown soil sulface temperature (°C), T_{old} is the calculated temperature of the second nodal point from the previous time step (explicit approximation) and L_1 is the length of the first element (the numbering is started from the soil surface). T_{surf} is the temperature that is used as an upper boundary condition in the heat balance equation and T_{surf} can be solved from (8):

$$T_{surf} = \frac{T_{old} + w_{surf} \cdot T_{air}}{1 + w_{surf}} \qquad (9a)$$

$$w_{surf} = \frac{K_{snow} \cdot L_1}{K_{soil} \cdot Z_{snow}} \qquad (9b)$$

Thermal conductivity of snow is strongly dependent on density and the formula suggested is (Corps of Engineers 1956):

$$K_{snow} = S_K \cdot d_{snow} \qquad (10)$$

where parameter S_K has a value of 2.86E-06 (W m^{-1} °C m^{-3}kg).

3. Testing of the simulation program

3.1 Backas experimental station

The Backas experimental field was situated about 20 km north of Helsinki (24°58' E and 60°17' N). The measurements were started on June 1940 and thee observation period was about two years. Juusela (1945) has described the measurement field in detail and published the results of the experiments.

The soil at the experimental field was sticky clay with a clay content of 55-65 % in the topsoil and about 75-85 % in the subsoil. In 1940-1942, soil temperature was measured at the following depths: 2.5, 5, 10, 15, 20, 30, 40, 50, 60, 80, 100 and 125 cm. Moreover, gravimetric moisture content measurements were made at five different depths (20, 40, 60, 80 and 100 cm) and groundwater level depth was measured at 16 observation points.

3.2 Determination of pF-curve, hydraulic conductivity curve and soil thermal conductivity

The saturated hydraulic conductivity was estimated using the method of Bloemen (1980) which is based on soil texture. The soil water retention curves and hydraulic conductivity values were not measured and it was necessary to estimate them based on the methods described by Karvonen (1988).

The effect of ice on the hydraulic conductivity is taken into account with the formula proposed by Motovilov (1977)

$$K_{I,r} = K_{o,r} \frac{1}{(1 + c_k \cdot I)^2} \qquad (11)$$

Where $K_{I,r}$ and $K_{o,r}$ are relative conductivities at ice content I and in unfrozen soil, respectively, with an average value equal to 8.

The results obtained at the experimental field Backas are ideally suited for the verification of the heat balance model due to the fact that soil temperature was measured at different depths as a function of time. The estimation of the soil thermal conductivity was based on a short observation period in 14-15.5.1941 (24 hours) when the soil temperature was measured every second hour. The thermal conductivity function was determined using a simplified Kalman filtering algorithm (Karvonen 1988). The general idea behind the application of the simplified Kalman filter is to fully utilize an intensive measurement period.

The simplified Kalman filter was used to estimate the soil thermal conductivity using measured soil temperature (at point B) at 2.5 cm as an upper boundary condition and measured soil temperature at 125 cm as a lower boundary condition. The intermediate soil temperature measurements at depths 5, 10, 15, 20, 30, 40, 50, 60, 80 and 100 cm were given as the measurements. Soil moisture was assumed to be constant during the 24 hours. According to the estimation procedure, soil thermal conductivity can be represented by a function

$$K_T(w) = 0.56 + 0.57\sqrt{w} \qquad (12)$$

3.3 Prediction of soil temperature, soil moisture and frost depth

When using the model for prediction the air temperature must be used as an upper boundary condition for snowfree periods and in the case that the soil is covered by snow, the upper boundary condition must approximated by a method that has been discribed in section 2.5 (equation 9). In order to be able to predict the soil surface temperature, a snowmodel must be included since the snowcover effectively dampens the large fluctuations of the air temperature (for details of the snowmodel see Karvonen, 1988).

The lower boundary condition of the heat balance equation causes some problems, as well. In this case the profile is extended to the depth of 7 m and it is assumed that the lower boundary is a no-flow boundary in the heat balance equation.

The lower boundary condition in the mass balance equation was a calculated flow rate towards the subsurface drains.

The results from numerical experiments have been presented in Fig. 1 for soil temperature, in Fig. 2 for soil moisture, in Fig. 3 for groundwater level depth and in Fig.

4 for frost depth and depth of snow.

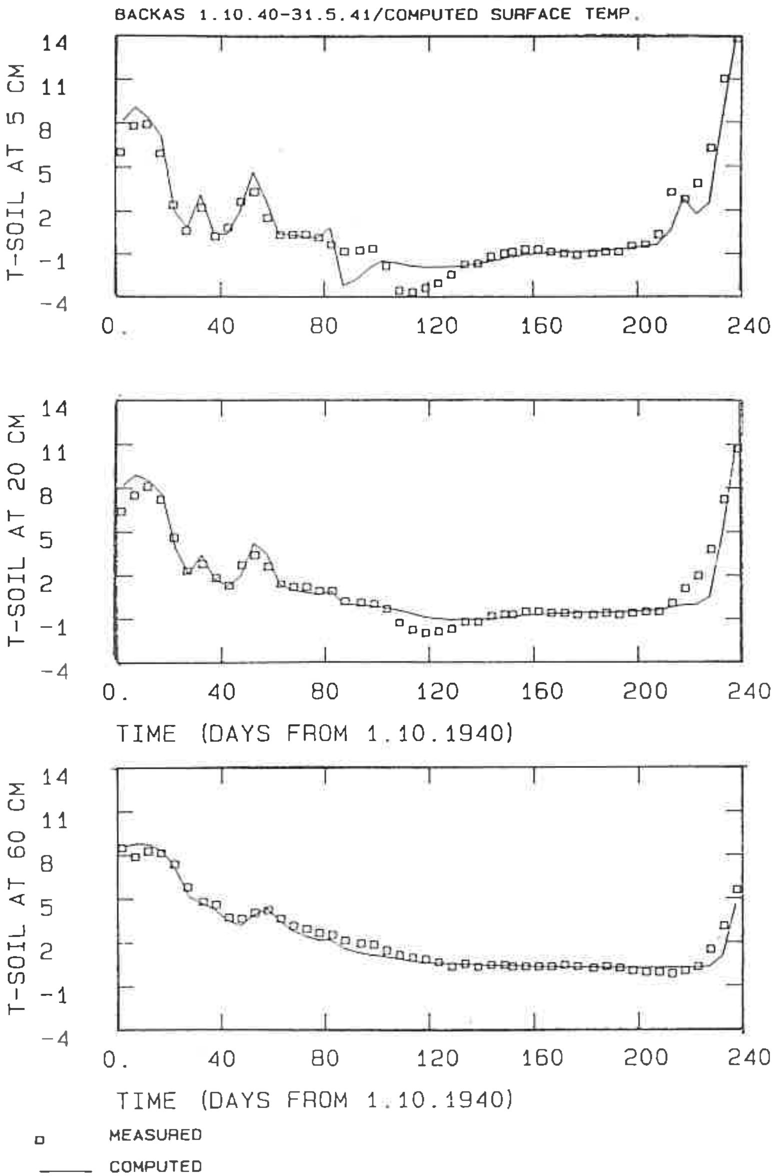

Fig. 1 Measured and computed soil temperature at four different depths in the case that soil surface temperature was calculated, a) 5 cm, b) 20 cm and c) 60 cm.

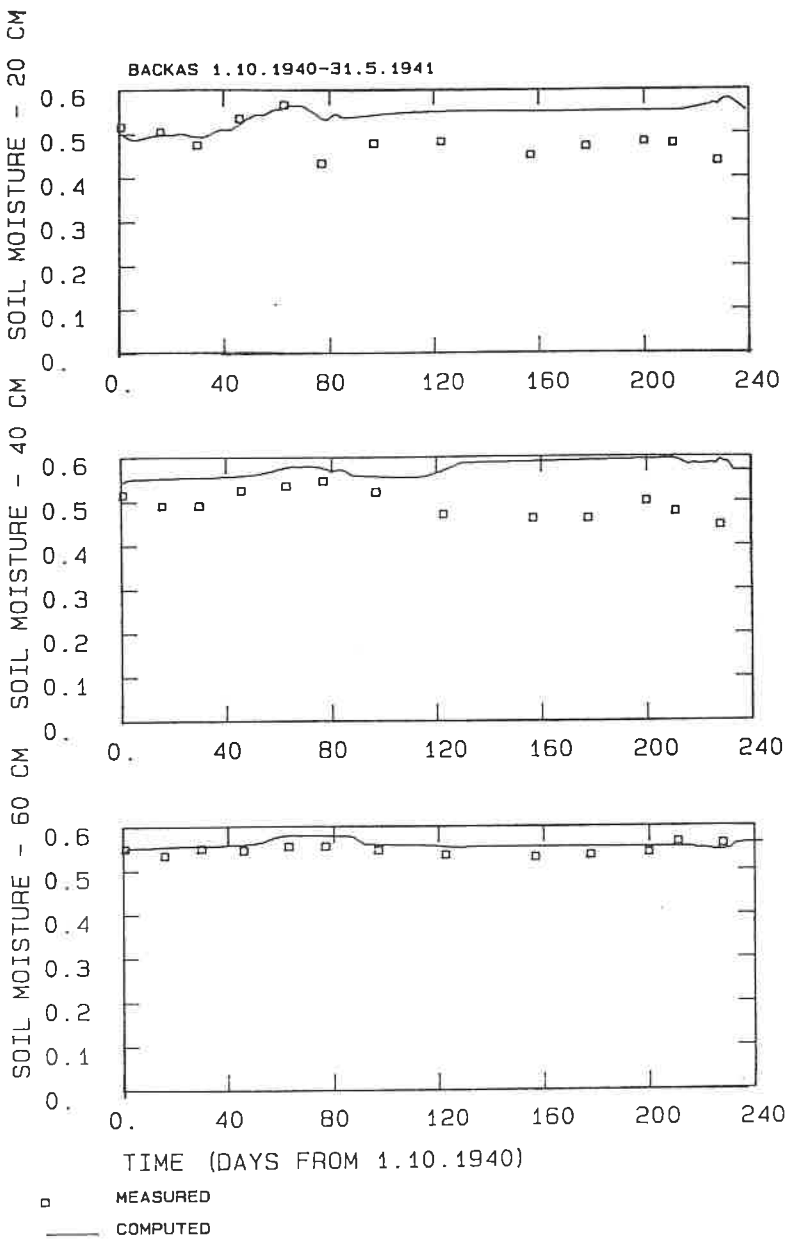

Fig. 2 Measured and computed soil moisture at three different depths a) 20 cm, b) 40 cm and c) 60 cm.

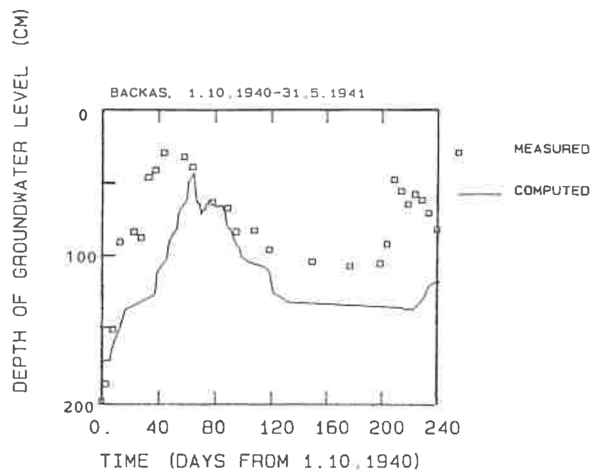

Fig. 3 Measured and computed depth of the groundwater level.

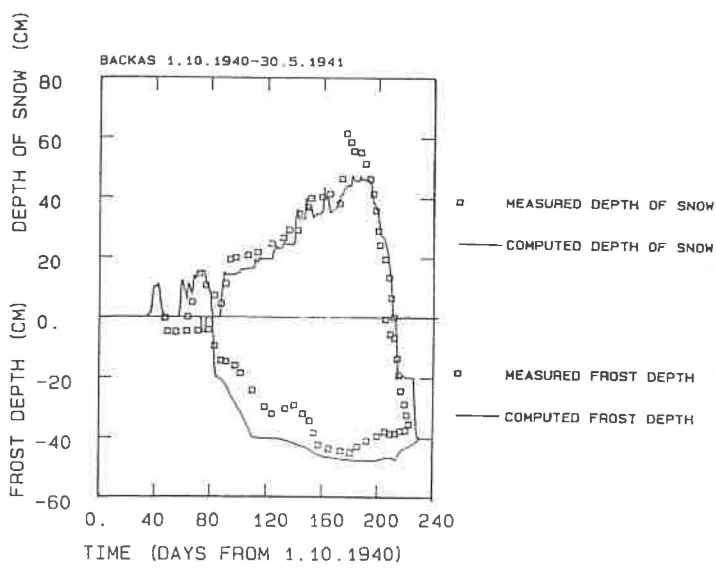

Fig. 4 Measured and computed depth and thickness of snow cover.

According to Figs. 1a and 4 it can be concluded that the computation of soil temperature is strongly influenced by relatively small errors in the predicted depth of snow cover. Computed soil temperature is about 1-2 °C too low for a period between t=83 d and t=100 (Fig. 1a). The main reason for the underestimation of the soil temperature during that time is that the computed snowdepth was equal to zero whereas the measure depth of snow was about 10 cm. The model assumed that the soil surface temperature was equal to air temperature (about -4 °C), but actually a higher boundary condition should have been used due to the existing snowcover.

The computed soil moisture content (Fig. 2) and predicted depth of groundwater level (Fig. 3) are in relatively poor agreement with the measured values. By calibrating the saturated hydraulic conductivity values, it would be possible to obtain better simulation results. However, there was not a suitable period of intensive measurements which could have been used in the simplified Kalman filter and therefore, the first estimates based on the method of Bloemen (1980) were accepted as final values for saturated hydraulic conductivity values.

It is useful to point out that the measured soil moisture content and observed depth of groundwater level were not in agreement with each. E.q. during the time t=40 d and t=60 d, the observed depth of groundwater level was about 30-40 cm. The measured soil moisture content at the depth of 40 cm was at the same time 52-53 % instead of the saturated value which was 58 %.

The predicted frost depth was almost all the time sligthly greater than the observed values. This is partly due to the fact that in the model the frost depth had to be estimated by locating the depth of the temperature value 0 °C whereas the measured depth was defined as a boundary where the soil was truly frozen (in a point where temperature is 0 °C, the soil is generally still unfrozen). There was a timing error of a few days in the predicted disappearence of soil frost. This is mainly due to the too high frozen moisture contentwhich requires considerable amount latent energy.

4 CONCLUDING REMARKS

The developed model gives results which are in a reasonable close agreement with field measurements. Numerical instability, which has often caused problems in the simulation attempts, has been avoided. More data, however, is needed to verify the combined heat and mass balance in seasonally frozen soil.

Lately an interactive version of the computer program has been worked out. It will be used for teaching the processes of the freezing and thawing of the soils. The program includes ease-to-use data input and various types of on-line help facilities. It is intended to be used in microcomputers having colored graphics and MS-DOS operating system.

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